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Grivaux, Julien (F-PARIS6)

Chern classes in Deligne cohomology for coherent analytic sheaves. (English summary)

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In this interesting article, the author develops a theory of Chern classes of coherent sheaves on compact complex manifolds, in situations where no global locally free resolutions are available.

The classical theory of Chern classes on smooth complex algebraic varieties defines the Chern classes of coherent sheaves using global locally free resolutions and the splitting principle. Since global resolutions are not available on general complex manifolds, new ideas are needed to define Chern classes in that setting.

On compact manifolds, the problem can be solved for singular cohomology by using the existence of locally free real analytic resolutions and for Dolbeault cohomology using the properties of the Atiyah connection. In the article under review, the author solves the problem in the setting of compact complex manifolds, for any cohomology theory satisfying a certain set of axioms. His main geometric technique is the passage to a bimeromorphic model where a given coherent sheaf can be made to sit in the middle of a short exact sequence, whose ends are a torsion sheaf and a locally free sheaf.

Damian Roessler

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.