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Topological properties of Hilbert schemes of almost-complex fourfolds (I).

(English summary)

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The punctual Hilbert scheme $X^{[n]}$ parametrizes the zero-dimensional subschemes of X of length n . The classical construction of $X^{[n]}$ when X is projective has been extended by C. Voisin [Ann. Inst. Fourier (Grenoble) **50** (2000), no. 2, 689–722; [MR1775365 \(2001k:32048\)](#); in *Symposium in Honor of C. H. Clemens (Salt Lake City, UT, 2000)*, 265–289, Contemp. Math., 312, Amer. Math. Soc., Providence, RI, 2002; [MR1941585 \(2004b:32045\)](#)] when X is an almost-complex compact four-manifold. The aim of the article under review is to extend some constructions and results related to the Hilbert scheme $X^{[n]}$ to the almost-complex case.

More precisely, it is shown that L. Göttsche's formula for the Betti numbers [Math. Ann. **286** (1990), no. 1-3, 193–207; [MR1032930 \(91h:14007\)](#)] and H. Nakajima's commutation relations for the creation/annihilation operators [Ann. of Math. (2) **145** (1997), no. 2, 379–388; [MR1441880 \(98h:14006\)](#)] hold in the almost-complex case. Tautological bundles associated with any complex bundle on X are constructed. *Laurent Evain*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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