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Automorphisms of rational surfaces with positive entropy. (English summary)

A smooth complex surface is said to be rational if it is birationally equivalent to
the projective plane $\mathbb{P}^2$. This paper involves the construction of automorphisms
of rational surfaces which have positive entropy. Infinite families of rational surfaces
carrying automorphisms of positive entropy have been discovered by two different
approaches. One approach was given by C. T. McMullen [Publ. Math. Inst. Hautes
gave an
exhaustive account of the possibilities for automorphisms produced by quadratic maps
with invariant curves. More recently T. Uehara ["Rational surface automorphisms with
positive entropy", preprint, arXiv:1009.2143] has considered a higher degree case.

The other approach was given by K. Kim and the reviewer [Michigan Math. J. 54
553–583; MR2496566 (2010g:37065)]. The second of these papers also showed that a
rational surface automorphism of positive entropy does not need to have an invariant
curve, and in fact the existence of an invariant curve seems rather special.

The paper under review presents a method for constructing rational surface auto-
morphisms which is somewhat related to the second approach mentioned above. This
approach starts with a well-chosen birational map $\Phi$ of $\mathbb{P}^2$. The objective is to find
$\varphi \in \text{PGL}(3, \mathbb{C})$ and an (iterated) blowup $\pi : X \to \mathbb{P}^2$ such that the lift $f_X$ of
$f := \varphi \circ \Phi$ will induce an automorphism of $X$.

In ["Transformations birationnelles de petit degré", preprint, arXiv:0811.2325], D.
Cerveau and J. Deserti gave a classification of the cubic birational maps of the plane,
modulo right and left action by linear transformations. Using this classification, the
authors select two cubic maps $\Phi_3$ and $f$ which are “promising” in the sense that
the exceptional locus is relatively uncomplicated. They apply their approach to show
that there are linear maps $L$ and $M$ such that $F_1 := L \circ \Phi_3$ and $F_2 := M \circ f$ both
produce rational surface automorphisms of the same positive entropy $\log((3 + \sqrt{5})/2)$. This subject has been treated further by J. Blanc ["Dynamical degrees of (pseudo)-

It has been noted by Kim and the reviewer [Math. Ann. 348 (2010), no. 3, 667–
688; MR2677899 (2012a:32014)] that there are nontrivial continuous families of rational
surfaces $X_\alpha$ and automorphisms $f_\alpha \in \text{Aut}(X_\alpha)$ with positive entropy. In fact, the
parameter $\alpha$ can be taken inside a space with arbitrarily large dimension, and it is shown
that the correspondence of dynamical systems $\alpha \to (X_\alpha, f_\alpha)$ is fully nontrivial, in the
sense of smooth conjugacy. The concept of nontriviality is that for distinct parameters
$\alpha_1$ and $\alpha_2$, the dynamical systems $(X_{\alpha_1}, f_{\alpha_1})$ and $(X_{\alpha_2}, f_{\alpha_2})$ are not conjugate. The
precise choices of conjugacy—topological, smooth, or holomorphic—will give different
concepts of nontriviality. The authors approach this issue by formalizing the more
abstract idea of deformation spaces of manifolds and automorphisms. They work with
holomorphic conjugacy and propose a method for determining the number of effective
parameters of a family.

Eric Bedford

References


*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*

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