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The Hochschild-Kostant-Rosenberg isomorphism for quantized analytic cycles.
 (English summary)

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The Hochschild-Kostant-Rosenberg (HKR) isomorphism identifies the Hochschild homology of finitely generated regular commutative k -algebras with their modules of Kähler differentials. It has many useful generalizations, e.g., in the context of derived category in algebraic geometry and higher algebra in stable homotopy theory.

In this paper, following Kashiwara's idea, the author constructs HKR isomorphisms associated with pairs (X, Y) of complex manifolds such that X is a closed complex submanifold of Y and the normal (or conormal) exact sequence associated with the cycle X is holomorphically split, which is equivalent to the injection of X into its first formal neighborhood \overline{X} admitting a holomorphic retraction. For any such retraction σ , the locally-free sheaf $\sigma^*N_{X/Y}$ is a quantization of $N_{X/Y}$. Then there exists an HKR isomorphism Γ_σ between the derived tensor product $\mathcal{O}_X \otimes_{\mathbb{L}\mathcal{O}_Y}^{\mathbb{L}} \mathcal{O}_X$ and $\bigoplus_i \wedge^i N_{X/Y}^*[i]$, independent of the retraction σ .

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