MR3228623 (Review) 14F43 14C40 32C35 58J52
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Variation of the holomorphic determinant bundle. (English summary)
Math. Res. Lett. **20** (2013), *no. 6,* 1091–1101.

The paper under review concerns the variation of the determinant bundle of the derived push-forward of a family of holomorphic vector bundles by a proper holomorphic submersive map between complex manifolds.

More precisely, let $f\colon X \to Y$ be a proper holomorphic submersion between complex manifolds and $\{\mathcal{E}_t \to X\}_{t\in\Delta}$ a family of holomorphic vector bundles parametrized by the complex unit disc.

The first result is the following "rational variation formula" in rational Deligne cohomology:

$$c_1(\det Rf_*\mathcal{E}_s) - c_1(\det Rf_*\mathcal{E}_t) = \left[f_*\big(\mathrm{ch}(\mathcal{E}_s) - \mathrm{ch}(\mathcal{E}_t)\big)\cdot \mathrm{Td}(T_{X/Y})\right]^{(2)},$$

for any $s,t \in \Delta$, where the superscript on the right-hand side stands for the degree two component of the Grothendieck-Riemann-Roch-type term considered. The reader can find also an "integral variation" version of the above formula, under some assumption of separateness for $H^1(Y,\mathcal{O}_Y)$ and of countability for $H^1(Y,\mathbb{Q}_Y)$.

Finally, as an application towards the general Grothendieck-Riemann-Roch formula in Deligne cohomology, the author shows the following result: in the case where $X = Y \times F$, with F compact, for any $\mathcal{L} \in \mathrm{Pic}^0(X)$ we have that

$$c_1(\det Rp_*\mathcal{L}) = p_*\big(c_1(\mathcal{L})\cdot [q^*\mathrm{Td}(F)]^{(\dim F)}\big)$$

in the second rational Deligne cohomology group of Y, where p and q are respectively the first and the second projection. *Simone Diverio*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.