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Loci in strata of meromorphic quadratic differentials with fully degenerate Lyapunov spectrum. (English summary)

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The article under review concerns the Lyapunov exponents of the Teichmüller flow. In a recent article [Publ. Math. Inst. Hautes Études Sci. **120** (2014), 207–333; MR3270590] A. Eskin, M. Kontsevich and A. Zorich have shown that the sum of the Lyapunov exponents $\lambda_1+\cdots+\lambda_g$ of an $SL(2,\mathbb{R})$ invariant suborbifold of a stratum of quadratic differentials can be related to the Siegel-Veech constant of the invariant locus. This sum can also be written as the integral over the invariant locus of the curvature of the Hodge bundle along Teichmüller disks [G. Forni, Ann. of Math. (2) **155** (2002), no. 1, 1–103; MR1888794]. It follows that every exponent λ_i is computed for cyclic covers of the sphere branched over four points.

The current article studies the phenomenon that for some cyclic covers the Lyapunov exponents sum to 0. The authors answer a question of Forni, C. Matheus and Zorich [J. Mod. Dyn. **5** (2011), no. 2, 285–318; MR2820563] as to whether this phenomenon occurs in other situations. They prove, in particular, that there exist closed $GL(2;\mathbb{R})$-invariant loci of quadratic differentials of arbitrarily large dimension with zero Lyapunov exponents. Also, the authors prove a general result relating zero Lyapunov exponents to the Teichmüller disk of a half-translation surface.

Theorem 1. Let (X,q) be a half-translation surface, n the number of poles of q, and $\mathbb{D}$ its Teichmüller disk. Then the following are equivalent:

(1) $\mathbb{D}$ lies in the Forni locus.
(2) The forgetful map $\mathcal{T}_{g,n}\to\mathcal{T}_g$ maps $\mathbb{D}$ to a point.
(3) For any (X_t,q_t) in $\mathbb{D}$, the extension of B_{q_t} to $\mathcal{Q}_t$ vanishes.
(4) All Lyapunov exponents of (X,q) are zero.

Here, B is Forni's B-form defined by

$$B_q(\alpha,\beta)=\int_X \alpha\otimes\beta\frac{|q|}{q}.$$

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.