This interesting paper gives a Lie theoretic study of the derived Ext algebra associated to a tame quantized cycle in a complex manifold or a smooth algebraic variety of characteristic 0.

Let me explain the background, following the introduction of this paper. The Hochschild-Kostant-Rosenberg (HKR) isomorphism has a version in the level of derived categories, which this paper calls the geometric HKR isomorphism. Let $X$ be a smooth scheme or a complex manifold. Then there is an isomorphism of the sheaf of polyvector fields on $X$ and the derived Hom sheaf of the diagonal of $X$. It is actually an isomorphism of algebras with respect to the wedge products and the Yoneda products. This algebra isomorphism is given by the composition of the global cohomological HKR isomorphism with the twist by the square root $\sqrt{\text{Td}(X)}$ of the Todd class.

One can reformulate this fact in terms of Lie theoretic language (see Theorem 5.9 for details): the shifted tangent sheaf $T_X[-1]$ is a Lie algebra object in the derived category $D^b(X)$, the universal enveloping algebra $U(T_X[-1])$ is the derived Hom sheaf, and the geometric HKR isomorphism coincides with the PBW isomorphism. Moreover the algebra isomorphism, which was a twist of the HKR isomorphism by $\sqrt{\text{Td}(X)}$, coincides with a Duflo isomorphism for the Lie algebra $T_X[-1]$.

The main body of this paper proposes an extension of the above Lie theoretic description of the geometric HKR isomorphism for a smooth scheme $X$ to the setting of a pair $X \subset Y$ of a smooth scheme $Y$ and a cycle $X$. In a Lie theoretic viewpoint, one is considering a pair $h \subset g$ of Lie algebras. Precisely speaking, one needs the condition called “tame quantized” for the cycle $X$. This condition has a natural Lie theoretical interpretation that $g = h \oplus m$ as $h$-modules and that $[\pi_h(m, m)] = 0$.

The main Theorem A states that the shifted normal sheaf $N_{X/Y}[-1]$ is a Lie algebra object in $D^b(X)$, and the associated universal enveloping algebra has an explicit description.

Theorem B states that the quantized cycle class $(X, \sigma)$ (see §4.3.1 for details) is the Duflo element of the Lie algebra $N_{X/Y}[-1]$. This result was originally found by S. Yu [Adv. Math. 352 (2019), 297–325, doi:10.1016/j.aim.2019.06.003], although a more Lie theoretic proof is given in this paper.

The text is clearly written and the introduction is very readable. I recommend this article to those interested in the representation theoretic study of derived categories of varieties.

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References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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