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Infinitesimal deformations of rational surface automorphisms. (English) Zbl 06860655

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The basic problem of deformation theory in algebraic geometry involves watching a small deformation of a family of objects, such as manifolds or schemes with extra additional structure or morphisms. . . .

In the paper under review, the author develops this theory for pairs (X, f) where X is a complex compact manifold and f is a biholomorphism of X . When X has a nonzero vector field, then by Kuranishi's theorem, X admits a universal deformation (\mathcal{K}, B_X) , where \mathcal{K} is the Kuranishi family and B_X is the Kuranishi space. The biholomorphism f of X acts naturally on B_X and the restriction of the family \mathcal{K} to the fixed locus Z_f is universal for the deformation functor of pairs (X, f) .

Precisely, the Zariski tangent space of Z_f at the origin is identified with fixed vectors in $H^1(X, TX)$ under the action of f_* . The author deduces a bound of the number of moduli of (X, f) . He proposes a computation of this action in various examples for rational surface automorphisms.

Reviewer: Mohamed Selmi (Sousse-Riadh)

MSC:

32G05 Deformations of complex structures

32M05 Complex Lie groups, automorphism groups acting on complex spaces

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