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Tian's invariant of the Grassmann manifold. (English) Zbl 1104.53067

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For a compact Kählerian manifold M, Tian's invariant $\alpha(M)$ is connected to the existence of Einstein-Kähler metrics [*G. Tian*, Invent. Math. 89, 225–246 (1987; Zbl 0599.53046)]. Tian established that if $\alpha(M) > m/(m+1)$, $m = \dim M$, then there exists an Einstein-Kähler metric on M. This condition is not necessary since, e.g., for the complex projective space $\mathbb{P}^m(\mathbb{C})$, $\alpha(\mathbb{P}^m(\mathbb{C})) = 1/(m+1)$.

In the article under review, for the complex Grassmannian manifold $G_{p,q}(\mathbb{C})$, $\alpha\big(G_{p,q}(\mathbb{C})\big) = 1/(p+q)$ is calculated, which is a generalization of Tian's result. At first, the volume element of the Kähler metric form $G_{p,q}$ is derived. Next, some general results concerning Tian's invariant as well as imbeddings of the product $\{\mathbb{P}^1(\mathbb{C})\}^p$ in $G_{p,q}(\mathbb{C})$, which allows the author to deduce $\alpha\big(G_{p,q}(\mathbb{C})\big)$ from $\alpha\big(\mathbb{P}^1(\mathbb{C})\big)$.

Reviewer: Zbigniew Olszak (Wrocław)

MSC:

53C55 Hermitian and Kählerian manifolds (global differential geometry)
53C25 Special Riemannian manifolds (Einstein, Sasakian, etc.)
32M10 Homogeneous complex manifolds

Keywords:

Kähler manifold; Einstein-Kähler metric; first Chern class; Tian's invariant; Grassmannian manifold

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