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Grivaux, Julien

Chern classes in Deligne cohomology for coherent analytic sheaves. (English) Zbl 1193.14026

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Let X be a smooth complex compact manifold. For locally free sheaves on X, there is a standard way to define Chern classes in a suitable cohomology theory, for example through the projective bundle formula. In a standard manner, Chern classes are also defined for coherent sheaves with locally free resolutions.

However if X is not algebraic, in general there is no locally free resolution of a coherent sheaf $\mathcal{F}$ on X. In this paper, the author addresses this problem by defining Chern classes, or equivalently Chern characters, for any coherent sheaf $\mathcal{F}$ using induction on the dimension of X.

The target for the author's Chern character ch can be any cohomology theory satisfying some axioms, the most important of which is the existence of Gysin maps for proper holomorphic maps. For example, rational Deligne cohomology satisfies these axioms.

In more detail, the author assumes that ch has been defined in dimension $n-1$ which satisfies all the desired properties. He then goes on to define ch in dimension n, and verify the desired properties in this dimension. The definition in dimension n breaks into several steps. First the author deals with torsion sheaves, that is, sheaves supported on divisors. Here he can invoke the induction hypothesis and build the Grothendieck-Riemann-Roch theorem in. Hironaka's resolution of singularity comes in to reduce the general case to a divisor with simple normal crossings. Then the author addresses sheaves which are locally free modulo torsion, which account for all sheaves after blow-ups. At various stages, he has to check compatibility and to verify the desired properties.

At the end, the author shows uniqueness of Chern classes satisfying some basic properties.

Reviewer: Zhaohu Nie (Altoona)

MSC:

14F43 Other algebro-geometric (co)homologies
14F05 Sheaves, derived categories of sheaves, etc.
19L10 Riemann-Roch theorems, Chern characters

Keywords:

coherent sheaves; locally free resolution; torsion sheaves; Chern character; Chern classes; blow-ups; Grothendieck-Riemann-Roch theorem; Whitney formula

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