

**Grivaux, Julien**

**Topological properties of Hilbert schemes of almost-complex four-manifolds. II.** (English)

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In the paper under review, the author studies the rational cohomology rings of Voisin's Hilbert schemes  $X^{[n]}$  associated to a symplectic compact four-manifold  $X$ . The author proves that these rings can be universally constructed from  $H^*(X, \mathbf{C})$  and  $c_1(X)$ , that Ruan's cohomological crepant resolution conjecture holds for  $X^{[n]}$  if  $c_1(X)$  is a torsion class, and that the complex cobordism class of  $X^{[n]}$  depends only on the complex cobordism class of  $X$ .

These results are straight-forward generalizations of known results when  $X$  is a smooth projective algebraic surface [*G. Ellingsrud, L. Göttsche and M. Lehn*, J. Algebr. Geom. 10, No. 1, 81–100 (2001; Zbl 0976.14002); *W.-P. Li, Z. Qin and W. Wang*, J. Reine Angew. Math. 554, 217–234 (2003; Zbl 1092.14007)].

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**MSC:**

14C05 Parametrization (Chow and Hilbert schemes)

14J35 Algebraic fourfolds

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Hilbert schemes; almost-complex four-manifolds; complex cobordism class

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