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Topological properties of Hilbert schemes of almost-complex fourfolds. I. (English)

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Given a smooth irreducible complex projective surface X, the Hilbert scheme $X^{[n]}$ of n points on X can be seen as a smooth resolution of the n-th symmetric product of X. Many topological properties of $X^{[n]}$ are known. The Betti numbers have been calculated by *L. Göttsche* [Math. Ann. 286, No. 1–3, 193–207 (1990; Zbl 0679.14007)] and only depend on the Betti numbers of X. This result has been clarified by *H. Nakajima* [Ann. Math. (2) 145, No. 2, 379–388 (1997; Zbl 0915.14001)] by constructing a representation of a Heisenberg algebra built from the rational cohomology of the surface on the direct sum $\mathbb{H} := \bigoplus \mathrm{H}^*(X^{[n]}, \mathbb{Q})$.

In the paper at hand, the author extends these results to Voisin's Hilbert schemes [*C. Voisin*, Ann. Inst. Fourier 50, No. 2, 689–722 (2000; Zbl 0954.14002)] associated to compact almost-complex four-manifolds. He is able to prove both Göttsche's formula and the defining commutation relations of Nakajima's operators in this context. One main ingredient of the proof is Le Poitier's decomposition theorem for semi-small maps (following the decomposition theorem by *A. A. Beilinson, J. Bernstein* and *P. Deligne* [Faisceaux pervers. Astérisque 100, 172 p. (1982; Zbl 0536.14011)] without using any characteristic p-methods or étale cohomology), which is included together with a proof as it is otherwise unpublished.

Finally, tautological bundles are defined in this almost-complex setting.

Reviewer: Marc Nieper-Wißkirchen (Augsburg)

MSC:

32Q60 Almost complex manifolds
14C05 Parametrization (Chow and Hilbert schemes)
14J35 Algebraic fourfolds

Keywords:

Hilbert scheme; Voison's Hilbert scheme; almost-complex four-manifolds; Göttsche formula; Nakajima operators

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