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Automorphisms of rational surfaces with positive entropy. (English) Zbl 1273.14029

The authors identify a systematic means of constructing a family of rational surface automorphisms from a birational map on $\mathbb{P}^2$. Specifically, the authors give sufficient conditions for a birational map of the form $\phi \circ f$, where $\phi$ is an automorphism of $\mathbb{P}^2$ and $f$ is a birational map on $\mathbb{P}^2$, to lift to an automorphism of a blow-up of $\mathbb{P}^2$, and then observe that holomorphic deformation of $\phi$ within the automorphism group of $\mathbb{P}^2$ may give a (positive-dimensional) holomorphic family of automorphisms associated to $f$. As noted in the paper, this methodology accounts for some previously studied examples of families of rational surface automorphisms with positive entropy. The bulk of the paper, then, is concerned with using the methodology to construct new examples of families of rational surface automorphisms with positive entropy; this process includes detailed constructions of blow-ups of $\mathbb{P}^2$ and local biholomorphisms between the blow-ups. The authors note that each new family introduced in the paper is locally holomorphically trivial, in the sense that locally deforming a map within the family yields maps that are conjugate by a holomorphic family of automorphisms of $\mathbb{P}^2$, and then observe that this property may be the expectation (though it certainly does not always occur) for families constructed via the methodology in the paper. The paper concludes with a general discussion about the number of parameters needed to describe a family of rational surfaces.

Reviewer: Paul Reschke (Ann Arbor)

MSC:
14E07 Birational automorphisms, Cremona group and generalizations
37B40 Topological entropy
32M99 Complex spaces with a group of automorphisms
14J26 Rational and ruled surfaces
14J50 Automorphisms of surfaces and higher-dimensional varieties
37F99 Complex dynamical systems

Keywords:
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