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Topological properties of Hilbert schemes of almost-complex four-manifolds II.
 (English summary)

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Summary: “In this article, we study the rational cohomology rings of Voisin’s Hilbert schemes $X^{[n]}$ associated with a symplectic compact four-manifold X . We prove that these rings can be universally constructed from $H^*(X, \mathbb{Q})$ and $c_1(X)$, and that Ruan’s crepant resolution conjecture holds if $c_1(X)$ is a torsion class. Next, we prove that for any almost-complex compact four-manifold X , the complex cobordism class of $X^{[n]}$ depends only on the complex cobordism class of X .”

{For Part I see [J. Grivaux, *Manuscripta Math.* **136** (2011), no. 1-2, 155–184; MR2820400 (2012j:14005)].}

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