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On Lang's conjecture for some product-quotient surfaces. (English summary)

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The geometric Lang-Vojta conjecture states that if X is a smooth projective surface of general type, then there are real numbers A and B and a proper Zariski closed subset $Z \subset S$ such that for any smooth projective curve and any holomorphic map $f\colon C \to X$ satisfying $f(C) \not\subset Z$,

$$\deg f(C) \leq A(2g(C)-2)+B.$$

The conjecture is still open even in the case of surfaces, although it has been proven for surfaces with $c_1^2 > c_2$ by F. A. Bogomolov [Dokl. Akad. Nauk SSSR **236** (1977), no. 5, 1041–1044; MR0457450].

The Green-Griffiths-Lang conjecture says that if a complex projective variety X is of general type, there exists a proper Zariski closed subset $Z \subset X$ such that for any non-constant holomorphic map $f\colon \mathbb{C} \to X$, one has $f(\mathbb{C}) \subset Z$ [cf. M. L. Green and P. A. Griffiths, in *The Chern Symposium 1979 (Proc. Internat. Sympos., Berkeley, Calif., 1979)*, 41–74, Springer, New York, 1980; MR0609557; S. Lang, Bull. Amer. Math. Soc. (N.S.) **14** (1986), no. 2, 159–205; MR0828820]. This conjecture has been proven for surfaces with $c_1^2 - c_2 > 0$ by Bogomolov (citation above) in the algebraic case and later by M. McQuillan [Inst. Hautes Études Sci. Publ. Math. No. 87 (1998), 121–174; MR1659270] in the holomorphic case.

In the present paper it is shown that both conjectures hold for a certain type of surfaces: product-quotient surfaces of general type with $p_g = 0$ and $c_1^2 - c_2 = 0$ (i.e., $p_g = 0$ and $c_1^2 = 6$). A product-quotient surface is the minimal resolution of singularities of a quotient $S = (C_1 \times C_2)/G$, where C_1, C_2 are smooth projective curves of genera $g_1, g_2 \geq 2$ and G is a finite group acting faithfully on each factor and not exchanging factors. The systematic study of product-quotient surfaces with $p_g = 0$ was started by I. C. Bauer and F. Catanese [in *The Fano Conference*, 123–142, Univ. Torino, Turin, 2004; MR2112572] and after work by several authors, the full classification was achieved in [Math. Comp. **81** (2012), no. 280, 2389–2418; MR2945163] by Bauer and R. Pignatelli.

The elaborate proofs of the results in this paper are obtained by using the classification of these surfaces and exploiting finely their particular geometry. In particular it is shown that the cotangent bundle is big, which allows the authors to use a result of Bogomolov.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.