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Derived intersections and free dg-Lie algebroids. (English. English summary)

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The paper under review gives a clean and conceptual approach to the results of the same author in [Adv. Math. **361** (2020), 106924; MR4041202], using the theory of free dg-Lie algebroids for derived connections introduced by M. M. Kapranov in [Selecta Math. (N.S.) **13** (2007), no. 2, 277–319; MR2361096].

Section 2 carefully sets up the notation and machinery for dg-Lie algebroids. Section 3 then discusses Atiyah bimodules, the essential tool for studying the objects at hand. Sections 4 and 5 contain the main results. These consist of

- a structure theorem for universal enveloping algebras and jet algebras of free dg-Lie algebroids;
- the existence of a cohomology class whose vanishing measures the existence of a derived connection;
- formality theorems for the universal enveloping algebra and jet algebra in terms of the vanishing of the aforementioned cohomology classes;
- a description of the the universal enveloping algebra and jet algebra in terms of a naturally defined square-zero extension; and
- an extension theorem for objects to extend to the square-zero extension, in terms of the aforementioned cohomology class.

Section 6 discusses the global versions of the results, giving a conceptual description of the objects at hand in the setting of quantized cycles, and their interpretation in deformation theory.

The paper is detailed, thus making for an excellent introduction to the technicalities of the subject. *Pieter Belmans*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.